Abstract—Cognitive Radio (CR) is an effective way to improve the utilization of spectrum resources. One of the most important challenges in CR networks is how to sense the existence of a signal transmission. Thus, detecting the existence of the primary users (PUs), which is called spectrum sensing, is a fundamental task in CRs. In a conventional single threshold energy detection scheme, the higher the probability of detection, the better the PUs are protected, however; it gives higher probability of false alarm which means less chances the channel can be reused when it is available. This paper is concerned with the analysis of the secondary users’ (SUs) achievable throughput in a double-threshold sensing scheme. An iterative algorithm is proposed to obtain the optimal values of the sensing time and second detection threshold. The results show that significant improvement in the throughput of the SUs is achieved when the parameters for the number of sensing rounds and the sensing threshold are jointly optimized.

Index Terms—Cognitive radio, double-threshold sensing scheme, sensing-throughput tradeoff.

I. INTRODUCTION

Cognitive radio is a spectrum agile radio technology which enables unlicensed users to access licensed spectrum by rapidly and autonomously adapting operating parameters to changing environments and conditions, without causing interference to licensed users [1], [2]. Actually, with the emergence of new wireless applications and devices, the last decade has witnessed a dramatic increase in the demand for radio spectrum, which has forced government regulatory bodies, such as the Federal Communications Commission (FCC), to review their policies [3]. Recently, the Federal Communications Commission (FCC) passed the proposal on spectrum reuse, allowing unlicensed operation in the bands of the PUs, such as TV broadcast bands [4], [5].

In order to ensure the licensed users use of specific band, the CR user has to accurately detect whether current band is used by a PU [6], [7]. As a result, efficient spectrum sensing is critical in the coexistence of primary and SUs in licensed bands. Incorrect decisions at the sensing stage leads to two consequences:

1) Missed detection of active PUs and imposing inadmissible interference to them.
2) Missed transmission opportunity when a primary channel is idle.

Thus, sensing performance is evaluated in terms of two probabilities. One is the probability of detection, which is the probability of the presence of the PU being detected. Another is the probability of false alarm, which is the probability that the PU is detected as presence, but actually it is absent. These are functions of both the sensing threshold as well as the sensing duration. Thus there exists a tradeoff between sensing capability and achievable throughput for the SUs. A longer sensing time will improve the sensing performance; however, with a fixed frame size, the longer sensing time will shorten the allowable data transmission time of the SUs [5].

In [8], the author shows that through utilizing the system with double-threshold detection scheme, the collision probability between the CR user and the PU is considerably decreased. Moreover, the interferences on the PUs are fairly reduced.

Although some research activities have been conducted in double-threshold sensing scheme, but to the best of authors’ knowledge, the effect of both optimal sensing threshold and sensing duration on the achievable throughput of the CR network has not been covered in the literature to date of writing this paper. For example, the author in [9], maximizes the throughput of a cooperative CR network through optimizing the $k$ in a k-out-of-n fusion rule, while thresholds were kept constant.

The main contributions of this paper can be summarized as follows. First, we study the problem of sensing-throughput tradeoff in a double-threshold sensing scheme. Particularly, we are interested in the study of the two thresholds energy detection scheme to maximize the achievable throughput of the SUs, while the PUs are sufficiently protected. Later, We formulate an optimization problem using the sensing time and second energy detection threshold as the optimization variables. Through this work, we also propose an iterative algorithm to obtain both the optimal sensing time and second energy detection threshold. The results show that the double-threshold sensing scheme is truly the optimized version of the single-threshold detection strategy and through jointly optimization of the sensing parameters, it can even achieve higher throughput.

The rest of this paper is organized as follows. The system model is explained in Section II. In section III, we formulate the sensing-throughput tradeoff problem in double-threshold sensing scheme. Computer simulations and numerical analysis are provided in Section IV to show the performance of the double-threshold sensing scheme. Finally concluding remarks are given in Section V.
II. THE SYSTEM MODEL

In our model, we consider the scenario where each SU employs energy detection to detect the presence of the PUs. Also, to perform periodical spectrum sensing, the CR network is assumed to employ a frame structure with frame consists of two parts: sensing slot \( T_s \) and transmission slot \( T - T_s \).

A. Single-Threshold Detection

Denote \( H_0 \) and \( H_1 \) as the hypotheses of the absence and the presence of the PU, respectively. When the PU is active, the discrete received signal at the SU can be presented as [10]

\[
y(n) = hs(n) + u(n),
\]

which is the output under hypothesis \( H_1 \). When the PU is inactive, the received signal is given by

\[
y(n) = u(n),
\]

and this case is referred to as hypothesis \( H_0 \).

The objective of energy sensing is to decide whether \( H_0 \) or \( H_1 \) is true by sensing the energy of signal \( y(n) \). The test statistic of energy detector is calculated as

\[
Y = \frac{1}{m} \sum_{i=1}^{m} |y(i)|^2.
\]

Denote that \( s(n) \) represents the signal from the PU and is assumed to be an independent, identically distributed (i.i.d) random process with zero mean and variance \( \mathbb{E}[|s(n)|^2] = \sigma_n^2 \). The channel coefficient \( h \) is the complex gain of the sensing channel between the PU and the SU. However, in this paper to make the approach tractable, the channel gain will be assumed to equal 1 under hypothesis \( H_1 \) and 0 under hypothesis \( H_0 \). The noise \( u(n) \), is also assumed to be i.i.d circularly symmetric complex Gaussian (CSCG) noise with zero mean and variance \( \mathbb{E}[|u(n)|^2] = \sigma_u^2 \). As defined in our model, each medium access control (MAC) frame consists of two parts: A sensing slot \( T_s \) and one data transmission slot \( T - T_s \), while \( T \) is the frame time period. Denote \( m \) as the number of signal samples that are collected during each sensing round, which is the product of the partial sensing time, \( \tau_{s,n} \), and the sampling frequency \( f_s \), where \( \{n=1, 2, ..., N_s\} \).

\( N_s \) denote number of sensing rounds required to make the exact decision on the presence/absence of the PUs. In a single-threshold scenario, \( N_s \) is equal to one, as there is no uncertainty region which makes the decision making process complicated; However, in a double threshold scenario \( N_s > 1 \).

Since, both number of samples, \( m \) and sampling frequency, \( f_s \) are assumed to be identical in all sensing rounds, we have

\[
\tau_{s} = \frac{N_s m}{f_s}.
\]

Following the work in [11], if the PU is inactive (i.e., \( H_0 \)), \( Y \) follows a central chi-square distribution with \( 2m \) degrees of freedom. Otherwise, \( Y \) follows a non-central chi-square distribution with \( 2m \) degrees of freedom and a non-centrality parameter \( \lambda \). Where \( \gamma = \frac{\sigma_n^2}{\sigma_u^2} \) represents the average received signal-to-noise ratio \( (SNR_p) \) of the PU measured at the secondary receiver of interest.

\[
Y \sim \begin{cases} 
\chi_m^2 & H_0 \\
\chi_m^2(\lambda) & H_1.
\end{cases}
\]

Hence, the output statistics of the energy detector can be characterized as follows:

\[
\begin{align*}
&f_{Y|H_0}(y) = \frac{1}{2^m \Gamma(m)} y^{m-1} e^{-y} \\
&f_{Y|H_1}(y) = \frac{(1+\gamma)^{m-\frac{1}{2}(2m+1)}}{2^{m-1} \sqrt{\pi} \Gamma(m)} \left[ 1 - \frac{\Gamma(m-1, \frac{\gamma y}{m-1})}{\Gamma(m-1)} \right].
\end{align*}
\]

Then, we obtain the Cumulative Distribution Function (CDF) of the two probability density functions as follows:

\[
F_{Y|H_0}(y) = \int_{0}^{y} f_{Y|H_0}(y) \, dt = 1 - \frac{\Gamma(m, \frac{y}{2})}{\Gamma(m)},
\]

and similarly

\[
F_{Y|H_1}(y) = \int_{0}^{y} f_{Y|H_1}(y) \, dt
\]

\[
= 1 - \frac{\Gamma(m-1, \frac{y}{2})}{\Gamma(m-1)} + \left( \frac{1+\gamma}{\gamma} \right)^{m-1} \times \left( e^{-\frac{\gamma y}{2m} \times \frac{\Gamma(m-1, \frac{\gamma y}{m-1})}{\Gamma(m-1)}} - 1 \right).
\]

Two types of error are considered in the energy detection scheme. "error type 0", when the PU is absent; however, it is detected as present, and "error type 1", when the PU is present; however, it is detected as absent.

The total probability of error can be defined as follows:

\[
P_e = Pr(H_1, H_0).P(H_0) + Pr(H_0, H_1).P(H_1),
\]

where \( P(H_0) \) and \( P(H_1) \) are the channel utilization factors by cognitive and PUs, respectively.

With the energy detector and focus on the complex PSK signal, the average probabilities of detection and false alarm for a Single-Threshold scheme can be approximated by [12]

\[
P_d(\beta_1) = Pr(Y > \beta_1|H_0) = Q \left( \frac{\beta_1}{\frac{1}{\sigma_n^2} - 1} \sqrt{m} \right),
\]

\[
P_d(\beta_1) = Pr(Y > \beta_1|H_1) = Q \left( \frac{\beta_1}{\frac{1}{\sigma_n^2} - \gamma} - 1 \sqrt{\frac{m}{2\gamma + 1}} \right),
\]

Where \( Q(.) \) is the complementary Gaussian distribution function defined as

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp \left( -\frac{z^2}{2} \right) \, dz.
\]

For a target probability of detection, \( P_{tar} \), the detection threshold \( \beta_1 \), can be determined by

\[
Q^{-1}(P_{tar}) = \left( \frac{\beta_1}{\frac{1}{\sigma_n^2} - \gamma} - 1 \sqrt{\frac{m}{2\gamma + 1}} \right).
\]
considering (10) and (11) by detector chooses neither detector, in addition to a region of uncertainty where the

\[ P_{d} = 1 - Q\left(\frac{\beta_{2}}{\sigma_{u}} - 1\right)\sqrt{m} \]  

(17)

and

\[ P_{d} = 1 - Q\left(\frac{\beta_{1}}{\sigma_{u}} - 1\right)\sqrt{m} \]  

(18)

Same as Single-Threshold scheme, for a target probability of detection, \( P_{d}^{\text{tar}} \), the detection threshold \( \beta_{1} \), can be determined by

\[ \beta_{1} = \sigma_{u}^{2} \left[ \frac{2\gamma + 1}{m} Q^{-1}(P_{d}^{\text{tar}}) + \gamma + 1 \right] . \]  

(19)

Fig. 1: The illustration of the relationship between the \( P_{d} \) and \( P_{f} \) in a single-threshold sensing scheme.

\[ \beta_{1} = \sigma_{u}^{2} \left[ \frac{2\gamma + 1}{m} Q^{-1}(P_{d}^{\text{tar}}) + \gamma + 1 \right] . \]  

(14)

From (10) and (14), the probability of false alarm can be derived as follows

\[ P_{f} = Q\left(\frac{\sqrt{2\gamma} + 1}{m} Q^{-1}(P_{d}^{\text{tar}}) + \sqrt{m} \right) . \]  

(15)

Note, the minimum required sensing samples to comply with the target probability of detection, can be determined by considering (10) and (11)

\[ m_{\text{min}} = \frac{1}{\gamma^{2}} \left[ Q^{-1}(P_{f}) - Q^{-1}(P_{d}^{\text{tar}}) \sqrt{2\gamma + 1} \right]^{2} . \]  

(16)

B. Double-Threshold Detection

The double-threshold energy detector employs two thresholds which define the same hypotheses as the single-threshold detector, in addition to a region of uncertainty where the detector chooses neither \( H_{0} \) nor \( H_{1} \). But, instead, reports that it is unsure which hypothesis is true. The use of Double-Thresholds scheme allows the probabilities of false alarm and missed detection to be set arbitrarily low at the cost of an increased uncertainty region; this is the principle on which double threshold energy detection relies.

Similar to the Single-threshold scheme, the average probabilities of detection and false alarm for a Double-Threshold scheme are given, respectively

\[ P_{d}^{\text{tar}}(\beta_{2}) = \Pr(Y > \beta_{2}|H_{0}) = Q\left(\frac{\beta_{2}}{\sigma_{u}} - 1\right)\sqrt{m} \]  

(17)

and

\[ P_{d}^{\text{tar}}(\beta_{1}) = \Pr(Y > \beta_{1}|H_{1}) = Q\left(\frac{\beta_{1}}{\sigma_{u}} - 1\right)\sqrt{m} \]  

(18)

As shown in Fig. 2, increasing \( \beta_{2} \) can decrease false alarm probability, which, however, increases the number of required sensing rounds as the probability that the sensed signal’s energy level falls between the two thresholds is becoming larger. Hence, we proceed to study the impact of \( \beta_{2} \) on the average number of required sensing rounds by deriving the probability \( q \), that the detected signal energy falls between the two thresholds as follows [12].

\[ q = \left[ F_{Y|H_{1}}(\beta_{2}) - F_{Y|H_{1}}(\beta_{1}) \right] P(H_{1}) + \left[ F_{Y|H_{0}}(\beta_{2}) - F_{Y|H_{0}}(\beta_{1}) \right] P(H_{0}). \]  

(20)

Denote \( \bar{N} \), the average number of sensing rounds that need to be taken during spectrum sensing is given by,

\[ \bar{N} = \sum n \times q^{n-1} \times (1 - q) = \frac{1}{1 - q}. \]  

(21)

Considering (20), it is clear that there is a tradeoff between the number of sensing rounds and false alarm probability while the detection probability is lower-bounded.

For a target probability of detection, \( P_{d}^{\text{tar}} \), the detection threshold \( \beta_{1} \) can be determined by

\[ Q^{-1}(P_{d}^{\text{tar}}) = \left(\frac{\beta_{1}}{\sigma_{u}} - 1\right)\sqrt{m} \]  

\[ \beta_{1} = \sigma_{u}^{2} \left[ \frac{2\gamma + 1}{m} Q^{-1}(P_{d}^{\text{tar}}) + \gamma + 1 \right] . \]  

(19)

Based on (17) and (18), we can rewrite (20) as follows

\[ q = \left[ P_{d}(\beta_{1}) - P_{d}(\beta_{2}) \right] P(H_{1}) + \left[ P_{d}(\beta_{1}) - P_{f}(\beta_{2}) \right] P(H_{0}). \]  

(24)

III. SENSING-THROUGHPUT TRADEOFF

In the previous section, we tried to find the first detection threshold \( \beta_{1} \), based on the target probability of detection. Since we are interested to simultaneously study the effect of both sensing threshold and sensing duration on the secondary network performance, we formulate the problem based on achievable throughput of the secondary network as both
parameters of interest are taken into account. To this end, let’s denote \( C_0 \) [bits/s/Hz] as the throughput of the SU when it operates in the absence of PUs, while \( C_1 \) [bits/s/Hz] is the throughput when it operates in the presence of PUs. Then,
\[
C_0 = \log_2 (1 + SNR_s),
\]
and
\[
C_1 = \log_2 \left( 1 + \frac{SNR_s}{1 + SNR_p} \right),
\]
where \( SNR_s = \frac{P_s}{N_0} \) and \( SNR_p = \frac{P_p}{N_0} = \gamma \). The two terms \( P_p \) and \( P_s \) are the interference power of PU measured at the secondary receiver and the received power of the SU, respectively.

Based on whether the PU is active, the secondary network throughput falls in two following scenarios:
- Scenario 1: The PU is present, but the SU could not detect it. In this case the achievable throughput of the SU is \( T - \tau_s C_1 \).
- Scenario 2: The PU is absent and there is no false alarm generated by the SU. In this case the achievable throughput of the SU is \( T - \tau_s C_0 \).

So, we can define the two throughputs, respectively as follows,
\[
R_1(\beta_1, \tau_s) = \frac{T - \tau_s}{T} C_1(1 - P_d(\beta_1))P(H_1),
\]
and
\[
R_0(\beta_2, \tau_s) = \frac{T - \tau_s}{T} C_0(1 - P_f(\beta_2))P(H_0).
\]

From (27) and (28), the average throughput of the secondary network is given by
\[
R_{Tot} = R_1(\beta_1, \tau_s) + R_0(\beta_2, \tau_s).
\]

The objective of sensing-throughput tradeoff in a double-threshold sensing scheme is to jointly optimize the detection threshold, \( \beta_2 \) and the sensing time, \( \tau_s \), so as to maximize the available throughput of the SU while the detection probability is constant and predefined. Actually, due to dependency of the achievable throughput to both sensing duration, \( \tau_s \) and sensing threshold, \( \beta_2 \) and also their interdependency based on (24) and (21), a joint optimization should be performed to find the maximum achievable throughput. Mathematically, the optimization problem can be stated as
\[
\begin{align*}
\text{maximize :} & \quad R_{Tot}(\beta_2, \tau_s) \quad \text{(30)} \\
\text{subject to :} & \quad 1 - \frac{1}{N_s} \leq q, \quad 0 \leq \tau_s \leq T, \quad \text{(31)}
\end{align*}
\]

Considering (4), the objective function in (30) can be written as
\[
\begin{align*}
\text{maximize } & \quad T - \frac{\sum_{\beta_2} N_s^m}{f_s} \left[ C_1(1 - P_d(\beta_1))P(H_1) \\
& \quad + C_0(1 - P_f(\beta_2))P(H_0) \right]. \quad \text{(33)}
\end{align*}
\]

We observe that this problem can be considered in the concave optimization category. To this end, we present the following result, to take advantage of the concavity of (33).

**Lemma 1.** The objective function in (33) is concave for all \( \beta_2 \) and \( N_s \).

**Proof:** The proof is given in Appendix. \( \blacksquare \)

Now, we can form the Lagrangian function for our optimization problem as follows:
\[
L(\beta_2, N_s, \nu) = T - \frac{\sum_{\beta_2} N_s^m}{f_s} \left[ C_1(1 - P_d(\beta_1))P(H_1) \\
& \quad + C_0(1 - P_f(\beta_2))P(H_0) \right] + \nu \left( q + \frac{1}{N_s} - 1 \right), \quad \text{(34)}
\]

Where the Lagrangian multiplier \( \nu \) is nonnegative. According to Karush-Kuhn-Tucker (KKT) condition [13], we have the following equations:
\[
\frac{\partial L(\beta_2, N_s, \nu)}{\partial \beta_2} = 0, \quad \text{(35)}
\]
\[
\frac{\partial L(\beta_2, N_s, \nu)}{\partial N_s} = 0. \quad \text{(36)}
\]

Since the two equations are non-linear and its difficult to get explicit expressions in terms of \( \beta_2 \) and \( N_s \), we numerically solve the equations for a specific Lagrangian multiplier.

In what follows, we show updating the Lagrangian multiplier \( \nu \), through the iteration process. To this end, we employ the projected subgradient method as follows:
\[
\nu(t + 1) = \left[ \nu(t) - \alpha(t) \left( q + \frac{1}{N_s} - 1 \right) \right]^+, \quad \text{(37)}
\]

where \([x]^+ \) denotes the projection onto the nonnegative area. \( \alpha(t) \) is also a positive step size for the \( t \) times iteration. After each iteration, the Lagrangian multiplier \( \nu \) updates accordingly. The newly calculated number of sensing rounds and the second threshold results are taken into account for the
Algorithm 1: Find the $\beta_2$ and $N_s$ that maximize $R_{\text{Tot}}$

1) Initialization: $\nu \leftarrow 1$, $t \leftarrow 0$, $\alpha \leftarrow 1$, $N_s \leftarrow 1$, $\beta_2 \leftarrow \beta_1$

2) Repeat
   a) Update $\nu$ according to (37)
   b) Calculate $\beta_2$ and $N_s$ according to (35), (36)
3) Stop, when $|\nu(t+1) - \nu(t)| \leq \epsilon$
4) $t \leftarrow t + 1$
5) $\alpha \leftarrow \frac{1}{\tau}$

where $\alpha$ is the step size, and $\epsilon$ is a given small constant.

next iteration. This procedure will be executed iteratively until converge.

IV. PERFORMANCE EVALUATION

In this section, numerical results along with simulation are presented to verify the effectiveness of the double threshold detection scheme. The parameters used for simulation are same as the ones given in Table I. We set the frame duration to be $T = 10\text{ms}$ and the sampling frequency to be $100\text{kHz}$.

Fig. 3 plots the maximum achievable throughput for cognitive radio transmission versus $\delta$, the difference between two thresholds $(\beta_2 - \beta_1)$. It is evident in the figure that there is a tradeoff between the sensing parameters, as further increasing the second detection threshold value (larger $\delta$), corresponds to an decrement in terms of the achievable throughput. In other words, for each $P_{md}$ there is an optimal $\delta$ for which maximum throughput is achievable. Moreover, the concave-shaped throughput is also consistent with the conclusion in Lemma 1.

In Fig. 4, the probability of error $P_e$ from (9), is plotted versus the second detection threshold for both single and double threshold detection schemes. $P_e$ is important because it provides a measure of the detection errors $P_f$ and $P_{md}$. A high $P_f$ value means poor spectral utilization while large $P_{md}$ translates into more harmful interference to the PU. Comparing to the single threshold detection scenario, where $\delta = 0$ and the detection errors are mutually independent, we observe that in a double threshold detection scheme we can further decrease $P_e$ while taking $P_{md}$ as a constant. Based on $P(H_0)$ and $P(H_1)$ (channel utilization factors), $P_e$ in a single detection threshold scheme reaches a point where there is no more feasible improvement; however, in a double threshold detection scheme $P_e$ can get any low values at the cost of increasing the average number of required sensing rounds, $N_s$.

In Fig. 5, it is evident that double threshold detection scheme achieves a performance superior to the single threshold detection one. For each $P_d$, increasing $\delta$ up to its optimal point increases the achievable throughput; however, in both schemes, the throughput degrades by increasing the $P_d$.

V. CONCLUSION

In this paper, we have proposed an iterative algorithm to obtain the second sensing threshold and the number of required sensing rounds that maximizes the throughput of the SUs in a double threshold sensing scheme, subject to adequate protection to the PUs. We have shown that significant improvement in the throughput of the SUs has been achieved.
when both the parameters for the sensing rounds and sensing threshold are jointly optimized.

**APPENDIX**

**PROOF OF LEMMA 1**

**Proof:** To prove this result, we compute the Hessian matrix as

\[
\begin{bmatrix}
(T - \frac{N_s m_f}{f_s}) \left( -\frac{(\beta_2-m)}{m} \right) - \frac{2m}{f_s} & 0 \\
-\frac{2m}{f_s} & 0
\end{bmatrix}
\]  

(38)

where \( r = \frac{0.1410 e^{-\frac{(\beta_2-m)^2}{2mT}}}{\sqrt{mT}} \). We observe that the matrix is negative semidefinite for all values of \( \beta_2 \) and \( N_s \). As a consequence, it is implied that the problem in (33) is concave.

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